

MATHEMATICS, DIALECTIC AND THE GOOD
IN THE REPUBLIC VI - VII
INTRODUCTION

Plato first introduces the doctrine of recollection in the *Meno* (80d ff.) to solve the problem of how one can know that a proposition is true and thus save an argument from being inconclusive. There, he makes Socrates assert that he *knows* that there is a difference between correct opinion (ὀρθὴ δόξα) and knowledge (ἐπιστήμη):

That there is a difference between correct opinion and knowledge is not at all a conjecture with me, but something I would particularly assert that I knew. There are not many things of which I would say that, but this one, at any rate, I will include among those that I know (98b).

Socrates, however, claims that a true opinion like that of Meno's slave, namely that the square of the diagonal is twice the size of the given square, can be converted into knowledge by a longer course of questioning (85c). This process is later described as tying down the true opinion 'by reasoning out the cause' (αἰτίας λογισμῷ), which process is said to be recollection (98a).

In the *Phaedo*, Plato again resorts to the doctrine of recollection in an effort to establish the ante-natal existence of the soul, and explains that recollection *begins* with sense-experience. There, he makes Socrates assert that we derive all our notions or conceptions of Forms from no other source — to do so would be impossible — than from sight or touch or some other one of the senses:

καὶ τότε ὁμολογοῦμεν, μὴ ἄλλοθεν αὐτὸ ἐννεονηκέναι μηδὲ δυνατόν εἶναι ἐννοῆσαι, ἀλλ' ἢ ἐκ τοῦ ἰδεῖν ἢ ἅψασθαι ἢ ἐκ τίνος ἄλλης τῶν αἰσθήσεων (75a)¹.

Again, Plato seems to be distinguishing between knowledge properly so called and true opinion there, when at 76b he makes Socrates explain that having knowledge implies the ability to 'give account' (λόγον διδόναι) of what one knows.

In the *Republic*, however, while distinguishing sharply between knowledge and opinion, Plato does not explicitly mention the doctrine of recollection. However, as Adam² has noted, the doctrine that education consists not in putting knowledge into the mind as if one were putting sight into blind eyes, but in turning the mind already having latent knowledge in the right direction presents fundamentally the same view as that implied by the doctrine of recollection. I have argued elsewhere³ that in the Line passage Plato has all along at the back of his mind his description of the *levels of thought* or conditions of mind of the

1. Cf. also *Phaedo* 75e where Socrates is made to say: «But, I think, if we acquired knowledge before we were born and lost it at birth, and afterwards, by the use of our senses (ὅστερον δὲ ταῖς αἰσθήσεσι χρώμενοι), we regained the knowledge, would not the process which we call learning really be recovering knowledge which is our own?»

2. J. Adam, *The Republic of Plato*, ed. by D. A. Rees, Cambridge, 1963, Vol. ii p. 98.

3. 'A Theory of Mental Development: Plato's *Republic* V-VII', Part I, *Platon*, Vol. 28, 1976, pp. 288-300.

φιλοθεάμονες etc. and 'true philosophers' in terms of the metaphor of dreaming and being awake in relation to images and their originals at 476a ff., and that this idea of the mind waking up from an initial dream-like condition presupposes the doctrine of recollection. In the following discussion, I propose to show (i) that far from meaning that *διάνοια* is 'mathematical thought', and thus trying «to strike an unbridgeable distinction between the procedures of the mathematician and those of the philosophic dialectician»⁴, Plato means us to understand that while the process of acquiring knowledge of Forms or recollection begins at the mental stage of *διάνοια* with the use of the sensible particulars taken for granted at the mental stage of *πίστις* as images of Forms, the process cannot be completed at that stage when the mind dreams about the Forms, and (ii) that the method of *νόησις*, i.e. dialectic, is designed to awaken the mind from its dream-like condition and to complete the process. That is to say, the method of *νόησις* is designed to convert the hypotheses of *διάνοια* derived initially from sense-experience into knowledge 'by reasoning out the cause' — αἰτίας λογισμῶ.

I

MATHEMATICS

1. The use of sensible particulars in *διάνοια*

Plato begins his discussion of the methods of *διάνοια* and *νόησις* by drawing a contrast between them as follows :

... the mind, using as images the things which were previously imitated, is compelled to pursue its enquiry in one section (i.e. *διάνοια*) from hypotheses, not proceeding to a beginning, but to a conclusion; but in the other (i.e. *νόησις*) which leads to an unhypothesized beginning (*ἀρχὴ ἀνυπόθετος*), the mind proceeds from hypothesis without the images of the other, making its enquiry by means of Forms themselves, through Forms themselves (510 b).

Since one of Plato's main preoccupations in the Line passage is to assign grades of objects to the four subsections of the line, it is not uncommonly supposed that when he says that in *διάνοια* the mind employs the 'objects' of *πίστις* as images, he is saying, in effect, that the 'objects' of *διάνοια* are the originals of which those of *πίστις* are the images. As Ross⁵ puts it: «In the phrase, 'using as images the things which formerly were imitated' which tells us that the contents of the second subsection are the images of those of the third, as those of the first were images of those of the second, I find the clearest evidence that the equality of the two middle subsections of the line ... is something unintended». Naturally, this leads to the view that the objects in the subsection of *διάνοια* are Forms, and that the dianoietic mathematician is himself consciously aware of the fact that the subject matter of mathematics is Forms, and hence that he employs physical diagrams and models as aids to grasp the Mathematical Forms. I shall now proceed to show that we are not meant to understand that the mind of the dianoietic mathematician can grasp the Mathematician Forms or any other supra-sensible entities at this stage.

4. See K. M. Sayre, *Plato's Analytic Method*, (Chicago and London) 1969, p. 41.

5. David Ross, *Plato's Theory of Ideas*, Oxford, 1951, p. 47.

Socrates is made to explain the dianoietic mathematician's use of sensible particulars as images in the following passages :

(i) You know also that they make use of visible forms and make their arguments about them though they are thinking not about them, but about those things which they resemble, making their arguments for the sake of the square itself and the diagonal itself (510d).

(ii) The very things which they model and draw, of which there are shadows and reflections in water, these they use in their turn as images, seeking to see those very things themselves which can be seen only by thought (510e).

(iii) ... it uses as images the very things which are themselves imitated by those below them, and in comparison with those, they are esteemed as clear and placed in a separate division — τετμημένοις (511a).

Now, in passage (i) we are told that while making use of visible forms and making their arguments about them, the dianoietic mathematicians are really thinking about the 'the square itself' and the 'diagonal itself', i. e. the Forms. This passage is thought to be a very good piece of evidence for the view that the 'objects' of *διάνοια* are Mathematical Forms. As Robinson⁶ says: «Our expectation is strongly confirmed by the statement that the mathematician's interest is in 'the square itself' and 'the diagonal itself.'» Now, this seems to be something quite different from what Plato is saying in the passage. We are not meant to suppose that the dianoietic mathematician himself knows that he is really thinking about the Forms: they make use of visible forms and make their arguments *about them*, i. e. the visible forms (λόγους περὶ αὐτῶν ποιοῦνται). It is Plato who thinks that though they make their arguments about visible forms they are *really* thinking about Forms. Thus when he says in passage (ii) that the dianoietic mathematicians are seeking 'to see those things themselves which can be seen only by thought' — τῇ διάνοια,⁷ he is again speaking from the point of view of the 'true philosopher' who alone knows what the mind of the mathematician is *trying* to do at this stage. The dianoietic mathematician will not describe himself as making use of visible forms whilst really thinking about some supra-sensible entities more real than the figures he draws.

Any reader who obtains the impression from the text that the dianoietic mathematician is himself consciously aware of what he is doing is bound to be baffled when later on he finds Plato describing him as dreaming about reality, and as having 'opinion' (δόξα) and not knowledge properly so called (ἐπιστήμη)⁸.

6. Richard Robinson, *Plato's Earlier Dialectic*, Oxford 1953, p. 197.

7. The word *διάνοια* is here being used in its general sense of 'thought' and not in the specialized sense employed to designate specifically the third state of mind. Cf. οὐ κοῦν τούτου μὲν τὴν διάνοιαν ὡς γινώσκοντος γνώμην ἂν ὀρθῶς φαίμεν εἶναι, τοῦ δὲ δόξαν ὡς δοξάζοντος (*Rep.* 476d). Those things can indeed be seen only by thought; but surely Plato cannot mean that they can be seen by thought 'contaminated' by the senses as in *διάνοια*. See my article *loc. cit.*, p. 290.

8. ὄνειρώττουσι μὲν περὶ τὸ ὄν, ὕπαρ δὲ ἀδύνατον αὐταῖς ἰδεῖν (533b) ... ἀλλ' εἰ πῃ εἰδῶλου τινὸς ἐφάπτεται δόξη, οὐκ ἐπιστήμη, ἐφάπτεσθαι ... (*Rep.* 534c). A. S. Ferguson ('Plato's Simile of Light', Part II, *C. Q.* XVI, 1922, p. 28) seems to be hard put to it to explain away this application of δόξα and dreaming to the practitioners of the method of *διάνοια* on his view that the mind in *διάνοια* has some knowledge of Forms. Plato means that the level of thought or state of mind of the dianoietic mathematician is the same as that of the φιλοθεάμονες he has already described at 476a ff.

In effect, like the φιλοθεάμονες, the dianoietic mathematician does not have knowledge — no matter what he himself thinks; what he has is really opinion (δόξα), albeit a higher level of δόξα as compared with the δόξαι of εἰκασία and πίστις. For the purposes of his diagram (the divided line) however, Plato regards the δόξα of διάνοια as a lower grade of knowledge :

We have called them (i. e. dianoietic studies) knowledge from habit, though they really need some other name connoting more clearness than opinion and more obscurity than knowledge (533c-e).

Passages (ii) and (iii) seem to make it clear that we are not really meant to suppose that the only objects that can be used as images in διάνοια are physical diagrams and models, and hence that διάνοια is 'mathematical thought'. Plato seems to mean that though he did not mention diagrams and models as 'objects' of πίστις, they are ontologically the same objects as those he has already mentioned. In passage (ii) we are explicitly told that the objects employed in διάνοια as images are *those very objects* (αὐτά) which have their shadows and reflections in water. That is to say, any of the objects of πίστις can be employed as images by the mind at the stage of διάνοια. Again, in passage (iii) Plato seems to be saying that precisely because in διάνοια the mind employs the objects of πίστις as images in its reasoning *these very objects* are clearer in διάνοια than they are in πίστις. It would seem then that what Plato means is that whereas in πίστις the mind takes sensible particulars for granted as being originals, in διάνοια, by reasoning about these very objects, the mind is treating them as the *images of Forms* they really are, albeit unconsciously, and hence that διάνοια is a higher level of thought or state of mind than πίστις.

2. The Hypotheses of Διάνοια

The mind in διάνοια is said to be compelled (ἀναγκάζεται), while making use of sensible particulars as images, to pursue its enquiry from hypotheses, not proceeding to a beginning, but to a conclusion; and in νόησις too the mind is said to pursue its enquiry from hypotheses, but proceeds to a beginning (510b). Why is the mind said to be *compelled* to proceed the way it does in διάνοια? What precisely is the nature of these hypotheses? These questions have been much discussed, and widely different views have been expressed on them. In general many scholars feel that Plato is finding fault here with mathematics, or at least with the mathematicians of his day. As R. M. Hare⁹ puts it, «Plato's indictment of the mathematicians rests upon two main counts: that they use physical diagrams, and that in their studies the mind 'is compelled to make its enquiry starting from *hypotheses*, and proceeding not beginningwards but end-wards.' I shall now proceed to show that what Plato means is that, situated as we are, the only means whereby the mind can gradually recollect the Forms is to employ λόγοι derived initially from sense-experience as hypotheses.

Socrates is made to explain the dianoietic mathematician's use of hypotheses in the following passage :

I think you know that those who study geometry and arithmetic hypothesize the odd and the even, and the three kinds of angle, and other things akin to these

9. R. M. Hare, 'Plato and the Mathematicians', in *New Essays on Plato and Aristotle*, ed. by R. Bambrough, London, 1965, p. 21.

in each enquiry. Treating these as if they knew them (ὥς ταῦτα εἰδότες), and making them hypotheses (ποιησάμενοι ὑποθέσεις αὐτά), they do not think it necessary to give any account of them to themselves or to others, considering that they are obvious to everybody. Then starting from these (i. e. hypotheses) they go through the rest consistently and arrive at the conclusion they set out to investigate (510c-d).

On the strength of the phrases 'hypothesizing the odd and the even etc.' and 'making them hypotheses', it has been argued that the hypotheses of δῖα are not propositions at all, but rather notions of the nature of these objects¹⁰, or that the hypotheses are just these objects, namely, the odd, the even, the figures etc.¹¹ It is then argued that these are the 'objects' of δῖα. As R. M. Hare maintains, these objects are «a kind of surrogate or second class objects of knowledge, as if they, like the objects of the best sort of knowledge, were a class of things.»¹²

Now, indeed, to the 'true philosopher' the odd, the even, the square etc. far from designating objects which may be regarded as surrogate or suppositious Ideas truly designate Forms: the dianoetic mathematicians while making use of visible forms and making their arguments about them are *really* thinking about the Forms¹³ (510b). However Plato's language seems to indicate quite clearly that the mind at this stage *does not really know* what these Mathematical Forms are. The dianoetic mathematicians simply assume that they know them (ὥς εἰδότες), and do not consider it necessary to give any account of them, neither to themselves nor to others, considering that they are obvious to everybody — ὥς παντὶ φανερά (510c). Indeed, the level of thought or state of mind of these mathematicians is the same as that of the φιλοθεάμονες — sham philosophers, sophists, rhetoricians etc., who discourse about *things* like justice, goodness, beauty etc. without the slightest inkling that these things are supremely real and quite different from particular beautiful things etc.¹⁴ They are precisely the people who at 534b-d are contrasted with the philosophic dialectician, and described as being quite unable to fight their way through all *elenchi* determined to apply the test not of appearance and opinion, but of reality, and make their way to the end through all the *elenchi* without sustaining a fall in their discourse, and who for this reason know neither goodness itself nor any good thing.

If, as I have argued, Plato means that by reasoning about sensible particulars the mind is treating them as the images of Forms they really are, albeit unconsciously, then since we are told that while making its arguments about visible forms the mind is *really* thinking about the Forms (which it *does not yet know*), we must suppose that the mind derives its notions or conceptions of Forms from sense-experience; and since these notions or conceptions (νοήματα) are not

10. See R. S. Bluck, *Plato's Phaedo*, London, 1955, p. 162 ff.

11. See R. M. Hare, *op. cit.* pp. 22-24.

12. *Ibid.* p. 44.

13. τοῦ τετραγώνου αὐτοῦ ἕνεκα ... καὶ διαμέτρου αὐτῆς (510d). Note that 'the square itself' and 'the diagonal itself' refer to the Forms and not to entities intermediate between Forms and particulars.

14. J. Gosling, ('*Republic Book V: τὰ πολλὰ καλὰ κτλ.*', *Phronesis* V, 1960, pp. 120-121) rightly argues that these φιλοθεάμονες etc. who are also described as φιλόδοξοι (480a) are not really ordinary people, but rather men of learning who are likely to be mistaken for 'true philosophers' by the ordinary man.

really a grade of objects at all, it seems reasonable to suppose that the hypotheses of *διάνοια* are statements or proposition *involving notions or conceptions* of Forms derived from sense-experience. That is to say, to the 'true philosopher' the hypotheses of *διάνοια* are statements or propositions which *presuppose* the existence of Forms. Again, since Plato describes the dianoietic mathematicians as *wrongly* assuming that they actually know the odd, the even etc., he is speaking from the point of view of the philosophic dialectician when he describes their propositions as hypotheses. That is to say, these mathematicians will not describe themselves as proceeding from hypotheses any more than they will describe themselves as making their arguments about visible forms whilst really thinking about some supra-sensible entities more real than the particulars they employ.

It would seem then that these hypotheses constitute the 'knowledge' of *διάνοια*, and that to the philosophic dialectician they are really 'opinions' (*δόξαι*). It is these hypotheses or *δόξαι*, and not the objects in the subsection of *διάνοια*, that are said to be 'knowable if linked with a beginning' — *νοητῶν ὄντων μετὰ ἀρχῆς* (511d). Accordingly, Plato is saying in the *Line* precisely what he says in the *Meno* about the possibility of converting true opinions into knowledge. There, he tells us that true opinions have been stirred up in the mind of the slave *like a dream*, and that these true opinions are capable of being converted into knowledge (*Meno* 85c). Here in the *Republic*, Plato is saying, in effect, that the mind in *διάνοια*, by making use of the senses in its reasoning, can form true opinions about Forms the existence of which is presupposed by its hypotheses, but that dreaming as it does about reality, it cannot 'see' the Forms themselves with eyes awake, 'so long as it leaves its hypotheses undisturbed' (533c). Clearly then, it is the job of philosophical dialectic to awaken the mind from this dream-like condition by 'disturbing' these hypotheses or *δόξαι*, and to convert them into knowledge.

We are now in a position to see that Plato means that to obtain knowledge of Forms the mind is *compelled* to employ *λόγοι* derived initially from sense-experience as hypotheses. Now, the greatest obstacle to seeing that this is what Plato is saying in the *Line* passage is the presupposition that the upper section of the line is meant to be literally identified with the intelligible world of the *Sun* passage¹⁵. The upper section of the line is indeed the intelligible section; but it does not so much represent the intelligible world itself as the realm of knowledge. That world is accessible only to discarnate minds as we learn from the *Phaedo*. Plato means that, situated as we are, this is the only means whereby we can regain our knowledge of Forms, and that the process of acquiring knowledge of Forms or recollection begins at the stage of *διάνοια* with the use of sensible particulars as images of Forms. However, the mind in *διάνοια*, dreaming as it does about reality, does not feel called upon to go beyond its hypotheses to grasp the Forms; it rather regards them as pieces of knowledge — the indubitable premisses of all its demonstrations. If, however, the intelligible section of the line were really meant to be literally identified with the intelligible world, the mind

15. I have argued in an article, 'Διάνοια and the Images of Forms in Plato's *Republic* VI', *Platon*, Vol. 31, 1979 that the 'objects' of *διάνοια*, that is to say, the objects in that subsection of the line are meant to be ontologically the same objects as the 'objects' of *πίστις*, hence the equality of the two middle subsections of the line.

would not require the aid of sensible particulars, nor indeed of hypotheses, to attain knowledge of Forms; it would simply 'see' the Forms even at the stage of *διάνοια*.

It would seem then that Plato's description of the procedure of the mathematicians does not really constitute an indictment of mathematics as such; it does, however, constitute an indictment of the great majority of contemporary mathematicians, for Plato means that precisely because of their level of thought or state of mind they are quite incapable of completing the process of attaining knowledge properly so called. Thus we greatly misrepresent Plato if we say with Robinson that «Plato means ... that the mathematicians who usually proceed dogmatically, and occasionally hypothetically, ought to proceed always hypothetically, because they never know their ultimate premisses to be true;»¹⁶ for we are meant to understand that (i) the hypothetical treatment of the propositions of dianoietic mathematics belongs to philosophical dialectic, and that (ii) the dianoietic mathematician himself and all other *φιλοθέμουνες* are simply not ready at this stage in the development of their minds to pursue the study of philosophical dialectic which alone can lead the mind to the apprehension of the Forms which are the *nominata* of the terms of their hypotheses or propositions: *αὐτὸ δὲ κάλλος μήτε νομίζων μήτε ἄν τις ἡγῆται ἐπὶ τὴν γνῶσιν αὐτοῦ δυνάμενος ἔπεσθαι* (476c).

3. The Programme of Mathematical Education

When Plato illustrates the method of *διάνοιαι* with dianoietic mathematics, he is really anticipating his own programme of higher education in the ideal state. In the current state of affairs pupils at this stage embarked on a less systematic study of geometry, astronomy etc. concentrating on rhetoric, literary studies and what passed for philosophy. Plato, however, seems to consider that this type of higher education is misguided and injurious to the state, because it is mainly concerned with value Forms whose images are difficult to discern. Besides, those who obtain this type of higher education are apt to get away with the mistaken impression that they actually know what justice, goodness, beauty etc. are, and hence consider themselves fully qualified to be entrusted with political power. His own guardians will be confined at this stage to the systematic study of the mathematical sciences (525a-531c) which are concerned with Forms whose images are easier to discern, rather than with value Forms during this period of habituation.

Now, it is sometimes maintained that when at 525a ff. Plato discusses the way in which mathematics can serve as a useful propaedeutic to the study of philosophical dialectic, he means us to understand that this usefulness is dependent on the complete divorce of the mathematical sciences from sense-perception, and hence that what he is advocating in the *Republic* is 'the rationalistic or logistic theory of the nature of the mathematical sciences.'¹⁷ It is argued, in effect, that Plato in the *Republic* repudiates the use of sense-perception at any stage in the process of attaining knowledge of Forms. Thus Gulley maintains that «although the language of 'images' of Forms is prominent in the dialogue and although a theory of recollection is implicit in it, the doctrine of the *Republic* about the

16. *Op. cit.* p. 153.

17. See Norman Gulley, *Plato's Theory of Knowledge*, London, 1962, p. 57.

contribution of sense-perception to knowledge appears in many important respects to be opposed to what is implied in the *Phaedo's* theory of recollection... When he (Plato) describes the two levels of thought (i.e. πίστις and διάνοια) ... he does not recommend them as stages in the acquisition of knowledge, but criticizes them as those most commonly mistaken to be the levels of thought at which knowledge is acquired or becomes possible¹⁸.»

Plato, indeed, makes the study of mathematics an absolutely obligatory pro-
paedeutic to the study of philosophic dialectic; and he does explain why this should be the case. We are told (523a ff.) that one of the subjects to be studied is Arithmetic, and that it has the power to lead the mind away from the contemplation of things of sense towards reality, though 'no one makes the right use of it'. It is followed in order by Geometry (526c-527c), solid Geometry (527d-528e), Astronomy (528e-530c) and Harmonics (530c-531c). It is true that in each case Plato criticizes contemporary practice, but he does so not because of its use of physical diagrams or 'sensible images', but rather because of its failure to recognize and make proper use of the power of these images to lead the mind away from the sensible to the intelligible¹⁹. It is not the case that he condemns the use of diagrams and other sensible images as 'bad practice';²⁰ he rather means that for the training of the would-be philosopher good use must be made of the tendency of these images to prompt and facilitate abstract reasoning.

One of the passages which are usually cited to substantiate the view that Plato in the *Republic* repudiates the use of sense-perception at any stage in the process of acquiring knowledge of Forms is 528e ff.²¹, where Socrates is made to ridicule Glaucon for suggesting that it is quite obvious to everyone that the study of astronomy compels the mind 'to look upwards, and leads it away from things here to those higher things' i.e. the heavenly bodies, and to assert that «we shall pursue the study of astronomy as we do in geometry, by means of problems, and leave the things in the heavens alone.» However, that Plato cannot mean to suggest that in his ideal state his astronomers need not know where the various constellations are in the heavens, nor tackle in their studies problems that will enable them to explain the movements of the heavenly bodies, seems clear from the fact that he actually explains that these 'decorations' of the heavens are to be used 'as patterns for the study of those realities', i.e. Forms:

οὐκοῦν εἶπον, τῇ περὶ τὸν οὐρανὸν ποικιλίᾳ παραδείγμασι χρηστέον τῆς πρὸς ἐκεῖνα μαθήσεως ἕνεκα ... (529e)²².

18. *Ibid.* pp. 53-55. Cf. also p. 56 where he argues as follows; «It is quite wrong to interpret this part of the Line (i.e. διάνοια) as granting to mathematics a specially valuable status as the ideal intermediary between sensible and intelligible worlds, ideal in that its use of sensible 'images' is more efficacious in prompting the mind to recollect the Forms than is the case with sensible images of non-mathematical Forms.»

19. The proper thing to do is to employ these images, and ascend from these to problems. Cf. *Rep.* 531c.

20. Cf. Gulley, *op. cit.*, p. 56.

21. See especially Gulley, *op. cit.*, p. 57 ff.

22. Plato indeed is said to have set the Academy the problem of finding out on what hypotheses (τίνων ὑποθέσεων) the apparent irregularity of the movements of the heavenly bodies can be reconciled with the real regularity, so as to 'save the appearances'. See Simplicius, *de caelo*, 488, 21; 493, 31 (Heiberg).

Plato makes his meaning quite clear when in his discussion of Harmonics he makes Socrates say :

Their method corresponds exactly to that of the astronomers ; for the numbers they seek are those found in these audible concords, *but they do not ascend to problems* (ἀλλ' οὐκ εἰς προβλήματα ἀνίστασι) and the consideration of which numbers are inherently concordant ... (531c) ²³.

It is easy to be misled by what Plato is saying in this section of the *Republic* if one does not recognize that in the Sun, Line and Cave Plato is putting forward a theory of the development of the mind right from infancy — a theory which he also puts forward in the *Timaeus* (43a ff.) — and that the stages in this development of the mind are outlined in the Line and allegorically described in the Cave. That in this section of the *Republic* Plato is basing his educational programme on such a theory seems to be indicated by the fact that while at 532b ff. he says that the progress of the prisoners right from the very moment of unchaining to the viewing of the shadows and reflections in the outer world is the work of the mathematical sciences he has just outlined, he explains that dianoietic mathematics begins for his guardians at the age of twenty (537b), but that from childhood up to the age of seventeen or eighteen (536d—537a) they must have been introduced to these subjects, 'not in the form of compulsory instruction ... but in play' (536d-e). Then after two or three years of 'gymnastic' training too severe to combine with serious studies, a select number of twenty-years old will be promoted to undertake a more systematic study of the same mathematical sciences :

They will be required to bring together the studies they disconnectedly pursued as children in their elementary education into a comprehensive survey (σύνοψιν) of the relation of these studies to one another and to the nature of reality (537c).

Glaucon observes that this is the only way to make their learning securely established (βέβαιος), to which Socrates replies that it is also the chief test of the philosophic nature and its opposite, for 'he who can view things together is a dialectician, he who cannot is not' — ὁ μὲν γὰρ συνοπτικὸς διαλεκτικός, ὁ δὲ μὴ οὐ (537c).

Now, on the strength of this passage it is not uncommonly supposed that Plato means us to understand that between the ages of twenty and thirty his guardians will be pursuing their studies of the mathematical sciences in quite a different manner from that described in the Line passage ; in effect, that the guardians at this stage will dispense completely with the use of their senses like the philosophic dialectician. Thus Cornford ²⁴ says : «The whole of this passage is concerned solely with turning mathematics into a genuine science by reducing it from an assemblage of scattered theorems, or chains of theorems, resting on unproved but demonstrable hypotheses, to a single chain depending on a single principle.»

As I have argued, Plato means that the hypotheses or δόξαι of dianoietic mathematics can be converted into knowledge by means of the application of the method of νόησις which is said to proceed without the aid of sensible images. However, that Plato does not mean to suggest that at this stage his guardians will

23. At 530 Plato makes Socrates say : «as the eyes are framed for astronomy so the ears are framed for the movements of harmony».

24. F. M. Cornford, 'Mathematics and Dialectic in the *Republic* VI - VII' (*Mind*, XLI, 1932) in *Studies in Plato's Metaphysics*, ed. by R. E. Allen, London, 1965, p. 81.

be studying what may properly be described as philosophical mathematics seems quite clear when he makes Socrates proceed to explain :

It will be our task to make a selection of those who exhibit them best (i. e. the qualities of the dialectician) . . . and when they have passed the thirtieth year to promote them . . . to still greater honours, and to prove and test them by the power of dialectic to see which of them is able *to dispense with the eyes and the other senses*, and move on to reality itself accompanied by truth (537d).

That is to say, we are not meant to suppose that before the age of thirty the guardians will be required to study the mathematical sciences without making any use whatever of their senses, nor that before that age they can hope to attain knowledge of Mathematical Forms. On the contrary Plato means us to understand that the method which his guardians will employ at this stage is precisely the method of *διάνοια* which he has described in the Line passage. The conversion of the hypotheses of mathematics into knowledge belongs to philosophical dialectic; and since these hypotheses are initially derived from sense-experience it would seem that far from taking the logicist position associated with the names of Frege and Russell, Plato's position here, like that of Kant, is rather compatible with those of the formalist and intuitionist movements, both of which are opposed to the logicist programme of reducing mathematics to logic.

II

DIALECTIC AND THE GOOD

1. Philosophical Mathematics and Moral Philosophy

It is generally recognized that the hypotheses of dialectic are *λόγοι* or propositions assumed provisionally to be true; what does not seem to be clearly recognized is that the initial hypotheses of dialectic are precisely the hypotheses or *δόξαι* taken for granted as constituting pieces of knowledge at the mental stage of *διάνοια*. Hence Robinson's complaint: « . . . it would remain a very odd fact that Plato says mathematics uses hypotheses in a way which implies that he is going to say that dialectic does not, and then says that dialectic does so too.»²⁵ He adds: «This apparently false contrast probably really means that mathematics is condemned never to get beyond hypotheses to a 'beginning'; both methods start with hypotheses, but only dialectic ever reaches an irrefutable starting-point.»²⁶ This view clearly derives from failure to appreciate that Plato is describing a continuous process of acquiring knowledge. As I have argued, Plato means us to understand that the hypotheses or *δόξαι* of *διάνοια* become 'knowable when linked with a beginning' (511b). Once we recognize this we can readily appreciate that the method of dialectic is designed to complete the process of acquiring knowledge of Forms uncompleted at the mental stage of *διάνοια* when the mind dreams about Forms, and that in spite of Plato's insistence that dialectic proceeds without the aid of the senses, he cannot really mean to suggest that the use of sensible particulars 'as images' to obtain the initial hypotheses necessary for a dialectical enquiry is beneath the dignity of 'true philosophers'.

25. *Op. cit.*, p. 200. See also p. 154.

26. *Ibid.* p. 200.

Now, philosophical dialectic, as we have seen, begins for the guardians after they have attained the age of thirty, having undergone intensive training in the mathematical sciences for ten years. Considering the length of time to be devoted to these mathematical sciences, it seems reasonable to suppose that some at least of the guardians might begin at this stage to advance knowledge of these subjects²⁷. However, that Plato himself does not think so, seems to be indicated by the fact that he says it is after their thirtieth birth-day that the best of them will be promoted to the study of philosophical dialectic. It is only then that they will be required to dispense with their eyes and other senses, and 'move on to reality itself accompanied by truth' (537d). To appreciate what Plato is doing here it is absolutely necessary to see that the guardians are not supposed to be pursuing philosophical mathematics between the ages of twenty and thirty, and hence that they do not really get to know the Mathematical Forms at this stage. Failure to see this leads to the view that *διάνοια* is not really a distinct state of mind or level of thought. Thus Gulley says: «In many ways this attempt to maintain a distinction between mathematics and dialectic appears at this point to have lost the justification which it had in the Line, and to remain merely an artificial division to suit the plan of separating a programme of progressive education into distinct grades.»²⁸

If then the initial hypotheses of dialectic are the hypotheses taken for granted at the mental stage of *διάνοια*, then, since Plato says that his guardians will apply themselves to the sedulous practice of dialectic for five years after their thirtieth birthday (539d), and that thereafter they will be sent back into the cave to hold offices suitable to youth, *so that they may not be inferior to other people in experience* — *ἵνα μὴδ' ἐμπειρίᾳ ὑστερῶσι τῶν ἄλλων* (539e), it is reasonable to suppose that (i) from the age of thirty to the age of thirty-five the guardians will be engaged in the study of philosophical mathematics and not moral philosophy as it is generally supposed, (ii) during this period they will get to know the Mathematical Forms, (iii) they will not concern themselves with moral philosophy until after the age of thirty-five. If we require any further evidence for the view that Plato does not mean us to understand that his guardians will be engaged in the study of moral philosophy before they have acquired the *necessary experience* in public life it is that from 537e to 539d, he is actually engaged in criticizing the current practice of introducing young men to the study of moral philosophy; and he does regard his guardians at this stage as young men.²⁹

Plato apparently considers that having recollected the Mathematical Forms, his guardians only have to acquire the necessary experience of images of value Forms in public life to enable them recollect the value Forms themselves by means of the same dialectical procedure. As we are told in the *Meno*, since the whole of nature is akin, and the mind has learned all things, there is nothing to

27. Cf. F. M. Cornford, *op. cit.* p. 79, and pp. 80—85.

28. *Op. cit.* p. 58. Note that the mathematician at this stage is still said to be 'unable to give and receive account' (531e).

29. It would seem that Plato is here basing his objection to the practice of introducing young men to the study of philosophy on his theory of the development of the mind. However, Gilbert Ryle, (*Plato's Progress*, Cambridge, 1966, p. 155 ff.) considers that Plato's attitude here might be the result of a crisis in his life. Cf. also his 'Dialectic in the Academy', in *New Essays on Plato and Aristotle*, ed. by R. Bambrough, *loc. cit.*, p. 155 ff.

prevent a man who has recollected one single thing from discovering everything else (81 d). Thus we should expect that between the ages of thirty-five and fifty, while still holding public offices, the guardians will find time for the study of philosophical dialectic. As Socrates explains :

At the age of fifty those who have survived the tests and proved themselves altogether the best in every task and form of knowledge must be brought at last to the goal (540a).

This goal, as what follows clearly indicates, is knowledge of the Good. Thus Plato does not expect his guardians after twenty years of philosophical dialectic to have attained knowledge of the Good.

2. Knowledge of the Good

Now, it is generally supposed that the 'unhypothesized beginning' (ἀνυπόθετος ἀρχή) which is reached at the end of the 'upward path' of dialectic is the Good. Thus, it is supposed that knowledge of the Good is in some sense antecedent to knowledge of all other Forms, and indeed, that in the *Republic* Plato envisages the deduction of all knowledge properly so called from the Good. As Robinson puts it, in dialectic, the Good «is the presupposition of all categorical proof.»³⁰ I shall now proceed to show that this view of dialectic in the *Republic*, namely that the 'unhypothesized beginning', which is reached at the end of the 'upward path', is the Good, is a misunderstanding.

According to Robinson³¹ who says that «Plato does not explicitly say that the anhypotheton is the Good», our evidence for the view that Plato did think that there was really only one 'genuine beginning', namely the Good, «is indirect, for we infer it from the following facts» : (1) In the passage of the Sun, Plato has told us that the Good has a unique place in all our knowledge, and in the passage of the Line, he gives a unique place in our knowledge to the 'unhypothesized beginning'; (2) while the Line tells us that we reach the 'unhypothesized beginning' at the end of an upward path of reflection, the Cave says that the released prisoner sees the sun last of all, and here the sun probably means the Good as it did in the simile of the sun; (3) Plato in Book VII, while not explicitly mentioning the 'beginning' describes dialectic in language reminiscent of the Line, as pressing on until at last it reaches the Good: «When a man attempts by dialectic without any of the senses through the *logos* to press on to what each thing itself is, and does not desist until he grasps what the Good itself is by means of intelligence itself, he arrives at the end of the intelligible (532 A-B).»

Now (1) in the passage of the Sun, Plato does give a unique place in our knowledge to the Good, because the Good is said to be the cause of the existence of Forms in the intelligible world; but there is nothing in what he says in that passage to suggest that knowledge of the Good is antecedent to knowledge of all other Forms. Indeed, there, he explicitly describes the Good as being 'beyond being' — ἐπέκεινα τῆς οὐσίας (509c), which should rather suggest that the Good must be the *last* thing, and not the first, to be known.³²

30. *Op. cit.*, p. 167.

31. *Op. cit.*, pp. 159—160.

32. Plato actually says ἐν τῷ γνωστῷ τελευταία ἢ τοῦ ἀγαθοῦ ἰδέα καὶ μόγις ὁρᾶσθαι (*Rep.* 517b).

(2) In the allegory of the Cave, the sun represents the Good — it *does not* represent the Good in the analogy of the Sun; there, its rôle in the visible world is described as being analogous to the rôle of the Good in the intelligible world. The released prisoners, after viewing the shadows and reflections of physical objects in the outer world, do not lift up their eyes in an attempt to contemplate the sun in order that they may thereafter be able to see the objects around them; what they do, in fact, is to view the objects around them — i. e. the objects which represent Forms other than the Good, namely, the plants and animals etc., as well as the stars and the moon; later on, when they are ready to look the sun (i. e. the Good) full in the face, we are told that they get to know that «it is the cause of whatever is right in everything, while it is itself supreme in the intelligible world, and parent of reason and truth; without having had a vision of this Form, no one can act with wisdom, neither in his own life nor in matters of state» (517c). If Plato really thought that knowledge of the Good was antecedent to knowledge of all other Forms, we should expect him to say here something to the effect that «without having had a vision of this Form, no one can know anything.»

(3) When in the passage which Robinson quotes to substantiate the view that Plato has only one 'genuine beginning' in dialectic, Socrates says that the dialectician presses on 'to what each thing itself is' (ἐπ' αὐτὸ ὃ ἐστὶν ἕκαστος), and does not desist until he grasps what the Good itself is', we are surely meant to see that knowledge of the Good comes after other Forms have been known. The passage, in fact, does not tell us that the dialectician aims at grasping the Good, in order that he may proceed to deduce all other Forms therefrom; what it does tell us is that the dialectician strives hard to grasp 'what each thing itself is', before finally — and that too not without perseverance — he grasps 'what the Good itself is'. Surely, 'what each thing itself is' can only refer to Forms generally, just as 'what the Good itself is' refers to the Form of the Good.

Indeed, Socrates proceeds to say:

And do you not give the name of dialectician to the one who is able to give account of the reality of each thing . . . and is this not also true of the Good? — οὐκοῦν καὶ περὶ τοῦ ἀγαθοῦ ὡσαύτως; (534b).

This passage seems to indicate clearly that Plato did not think that all dialectical enquiries necessarily end with, or involve the grasping of, the Good. Nevertheless, on this passage, Robinson comments: «It must also be said on the other side that a later passage suggests that the Good is only one of the crowd of dialectic's objects (and the same with the Good, 534B). But on the whole the evidence seems sufficient.»³³ That is to say, the evidence he has thus far adduced in support of the presupposition that the 'unhypothesized beginning' in dialectic is identical with the Good, is, despite this 'later passage', quite sufficient.

As we have seen, while Plato tells us that dialectic begins after the age of thirty for his guardians, he says the best of them will be brought to the goal at the age of fifty:

They must be brought at last to the goal; we shall require them to turn upwards the vision of the mind and fix it on that which sheds light on all; and when they have thus seen the Good itself, they shall use it as a pattern for the right

33. *Op. cit.*, p. 160.

ordering of the state and the citizens and themselves throughout the remainder of their lives (540a-b).

Clearly then, if we insist on identifying the 'unhypothesized beginning' with the Good what we are doing, in effect, is to make Plato say that the 'upward path' of dialectic proceeds for twenty years before the Good is 'seen', whereupon the 'downward path' begins! ³⁴

It is not as if Plato himself does not explicitly say that in the realm of knowledge the Good is *the last and hardest thing to know* — ἐν τῷ γνωστῷ τελευταία ἡ τοῦ ἀγαθοῦ ἰδέα καὶ μάλιστα ὁρᾶσθαι (517b). Indeed, scholars are very much aware of this. However, it is thought that he cannot really mean it, since it is supposed that if anything is perfectly certain it is that Plato in the *Republic* envisages the deduction of the whole of knowledge from the Good. Thus in considering the question why Plato thinks that the dianoietic mathematicians do not really know their starting points, Robinson observes: «The context seems to suggest that it was his opinion at this time that nothing is really known unless it was deduced from the idea of the Good; and that perhaps was his opinion when he wrote the *Phaedo* too.» ³⁵

Now, if Plato does not mean us to understand that the 'unhypothesized beginning' reached at the end of the 'upward path' of every dialectical enquiry is the Good, what precisely constitutes an 'unhypothesized beginning'? If, as I have argued, the hypotheses of νόησις are λόγοι or propositions assumed provisionally to be true, then an 'unhypothesized beginning' is likely to be a proposition like the 'something adequate' (τὶ ἱκανόν) of the *Phaedo* — a proposition that is adequate in the sense that it cannot itself be established by a 'higher' proposition within the limits of the particular dialectical enquiry ³⁶. It is the 'beginning' of the whole — ἀρχὴ τοῦ παντὸς (511b), that is to say, it is the beginning, not indeed of the universe ³⁷, but rather of the whole of the particular dialectical enquiry. Again, since the job of dialectic is to convert the hypotheses or δόξαι of διάνοια into pieces of knowledge, it seems reasonable to suppose that in the course of the 'upward path' the dialectician is gradually recollecting the Forms presupposed by his hypotheses as he obtains hypotheses that are 'higher' in the sense that they are the *limiting conditions* for the truth of the 'lower' hypotheses. Thus he is making the hypotheses 'not beginnings but really hypotheses like steps and spring-boards' — οἷον ἐπιβάσεις τε καὶ ὁρμαί (511b). And if he is gradually recollecting the Forms, then he is, in effect, 'destroying the hypotheses' — τὰς ὑποθέσεις ἀναιροῦσα (533c) in the sense that he is gradually converting them into pieces of knowledge. By the time the dialectician arrives

34. Cornford (*op. cit.* p. 89) says: «The reader has not been prepared to find the vision of the Good separated in time from training in moral dialectic. In the programme of research there is, of course, no break.» This is presumably because of his view that «the results of this research ... would amount to a complete system of moral philosophy, securely deduced from the definition of Goodness» (*ibid.*, p. 88), while «the deduction of all mathematical truth already discovered from 'the existence of the One' could be set down in a continuous written discourse for students to study» (*ibid.*, p. 88).

35. *Op. cit.* p. 153.

36. See my article, 'The Rôle of the Hypothetical Method in the *Phaedo*, *loc. cit.*

37. David Ross (*op. cit.* p. 54) says: «The idea of the Good is not mentioned as such in the Line passage, but we can hardly doubt that 'the first principle of the universe' (511b 7) is an allusion to it.»

at the final proposition which is the 'unhypothesized beginning', he must be supposed to have completely recollected all the Forms necessary to guarantee the truth of that proposition. It is these Forms that the dialectician must grasp in order to confirm — ἵνα βεβαιώσῃται (533c) the truth of the final proposition, and with it the truth of the previous propositions. In this way true opinions are converted into pieces of knowledge 'by reasoning out the cause' — αἰτίας λογισμῷ (*Meno* 98a). It is only then that one can 'give account' (λόγον διδόναι) of a true opinion.

This means that the 'upward path' of dialectic here is identical with the 'upward path' of the hypothetical method described in the *Phaedo*³⁸, and that it is not a process of proof — it does not demonstrate the truth of the 'beginning'; this truth is guaranteed by the complete recollection of Forms possible only at the mental stage of νόησις and supervening at the end of the 'upward path' of dialectic. Thus the 'unhypothesized beginning' is not itself an object at all, though it is the result of the 'seeing' of objects namely the Forms involved in a particular dialectical enquiry.

Having grasped the 'unhypothesized beginning', the dialectician can then descend 'holding on to those things (i. e. propositions) which depend on it' — ἐχόμενος τῶν ἐκείνης ἐχομένων (511b). In this way, in the course of the 'downward path' of dialectic, other propositions not previously considered may conceivably be deduced, and some at least of the previous ones may be seen to require correction or more precise wording; and this the dialectician is now in a position to do, for having completely recollected all the Forms involved in the enquiry he now *knows* precisely what the terms of the propositions designate. Clearly then, the 'downward path' of dialectic here, like the proof of the immortality of the soul in the *Phaedo*, is not part of the hypothetical method at all; it is rather meant to be seen as a process of proof based on absolute incorrigible knowledge attained by means of the hypothetical method³⁹. Thus we should expect the accomplished dialectician who has attained knowledge of many Forms by means of the hypothetical method to concern himself thereafter mainly with activities connected with the 'downward path' of dialectic and the systematization of the various branches of knowledge⁴⁰.

3. Conclusion

It would seem then that Plato in the *Republic* envisages the axiomatization of the various branches of knowledge, not indeed with the Good, or any propo-

38. The method of διάνοια corresponds to the treatment of the initial hypothesis described in the *Phaedo* 100 a — 101 c and 101 d, and we should expect the initial hypotheses for non-mathematical dialectical enquiries to be treated in much the same manner before the 'upward path' begins. The entire process, as I have argued, is continuous.

39. I have argued this point in my article, 'The Rôle of the Hypothetical Method in the *Phaedo*', *Phronesis* (Forthcoming).

40. Hence Plato's preoccupation with the method of collection and division, which is a method for the exploration of the relationship between the various Forms, in later dialogues. That at the time of writing the *Republic* Plato himself was very much aware of the fact that the hypothetical method is not all that dialectic involves seems to be indicated by Socrates' refusal to describe further the *methods* of dialectic after he had described the hypothetical method twice: λέγε οὖν, τίς ὁ τρόπος τῆς τοῦ διαλέγεσθαι δυνάμεως καὶ κατὰ ποῖα εἶδη διέστηκεν, καὶ τίνες αὐτοῦ ὁδοί... (532d ff.).

sition about the Good, as a 'super axiom', but rather with a plurality of 'beginnings', and with each branch of knowledge having some 'beginnings' of its own — an idea which anticipates Aristotle's theory of science and Euclid's axiomatization of geometry. Knowledge of the Good itself, like knowledge of Beauty in the *Symposium*, comes much later in life, and is possible only after many branches of knowledge have been explored. When it does come, this knowledge comes as the greatest revelation in their lives, for they now realize that «it is the cause of whatever is right in everything, while it is itself supreme in the intelligible world, and the parent of reason and truth» (517c); and having seen it «they will use it as a pattern for the right ordering of the state and the citizens and themselves for the rest of their lives» (540b).

This, in effect, means that Plato does not really think that the Good has any rôle to play in the methodology of the Line; and if it has any epistemological significance at all apart from being an object of knowledge, it is presumably that knowledge of the Good strengthens the accomplished dialectician in his conviction that he has indeed been doing the right thing all along. He now knows for certain that all the other Forms as well as his own reason, owe their existence to the Good, which, as being the cause of the participation of particulars in Forms, makes knowledge possible⁴¹. It is the last and hardest thing to know in the realm of knowledge; and it is this knowledge that a guardian must possess if he is to be entrusted with political power presumably because Plato considers that it is this knowledge of the ultimate cause of all generation, existence and destruction that will enable his guardians to rule with uprightness and justice.

It would seem then that Plato's methodology in the *Republic* is based on the views about the nature of knowledge and the manner whereby it might be acquired expressed in the *Meno* and the *Phaedo*; ⁴² and it is in harmony with what he says in the *Symposium* about the process whereby knowledge of Beauty might be attained. There, it is particularly noteworthy that the process is said to begin in childhood with the experience of particular beautiful things; and knowledge of Beauty itself, which comes much later in life, is the result of a process of generalization repeated at progressively more abstract levels, sense-experience gradually ceasing to play a part in it (*Symp.* 210a–212a). Here in the *Republic* we are meant to understand that the process of acquiring knowledge of Forms begins with the use of sensible particulars as images of Forms, but that at a certain stage in its reasoning, the mind must be able to dissociate thought

41. Thus in spite of the mystical language employed to describe it, we should expect this knowledge of the Good to be the result of a dialectical enquiry, like Socrates' 'second voyage' in the *Phaedo*, into the nature of the cause of all generation, existence and destruction. This study of the Good, then, does not form part of the study of moral philosophy.

42. Robinson, however, argues as follows: «It seems reasonable to ask what it is in the nature of things and the nature of men that makes the hypothetical method desirable. For if a method is suitable, that must surely be because the reality sought to be known is such and such and the human mind that seeks to know it is such and such. That a method is good ought to be derivable from the situation to which it applies. But Plato's insight did not go as far as that.» (*op. cit.* p. 178). This, however, reckons completely without the theory of Forms and the theory of recollection.

completely from the senses if the Forms are to be 'seen'; this it can do only at the mental stage of νόησις — the level of thought or state of mind of the 'true philosopher'.

J. T. BEDU — ADDO

University of Cape Coast, Ghana

ΠΕΡΙΛΗΨΙΣ

Μαθηματικά, Διαλεκτική και τὸ Ἀγαθὸν εἰς τὴν Πολιτείαν VI — VII

Ἡ μεθοδολογία τῆς σειρᾶς, (εἰκασία, πίστις, διάνοια, νόησις), προϋποθέτει τὴν ἀρχὴν τῆς ἀναμνήσεως. Περιγράφων ὁ Πλάτων τὰς μεθόδους αἵτινες χρησιμοποιοῦνται ὑπὸ τοῦ πνεύματος εἰς τὴν «διάνοιαν» καὶ τὴν «νόησιν» δὲν ἐπιχειρεῖ νὰ ὑπογραμμίσῃ μίαν ἀγεφύρωτον διάκρισιν μεταξὺ τῶν διαδικασιῶν τοῦ μαθηματικοῦ καὶ ἐκείνων τοῦ διαλεκτικοῦ φιλοσόφου. Οὗτος ἔχει μᾶλλον εἰς τὸν νοῦν μίαν συνεχῆ διαδικασίαν ἐπιτεύξεως γνώσεως τῶν μορφῶν ἀρχομένην μὲ ἐμπειρικὴν αἴσθησιν κατὰ τὴν νοητικὴν φάσιν τῆς «διανοίας», καὶ ἀποκορυφουμένην, πολὺ βραδύτερον ἐν τῇ ζωῇ εἰς τὴν γνῶσιν τῶν μορφῶν, εἰς τὴν πνευματικὴν φάσιν τῆς «νοήσεως».

Ἡ «διάνοια» δὲν περιορίζεται εἰς τὴν «μαθηματικὴν σκέψιν». Διασαφηνίζων τοῦτο μὲ τὰ μαθηματικά ὁ Πλάτων προκαταλαμβάνει τὸ ἰδικόν του πρόγραμμα τῆς ἐκπαιδεύσεως εἰς τὴν ἰδανικὴν πολιτείαν του. Αἱ ὑποθέσεις, ἀμφοτέρων, καὶ τῆς «διανοίας», καὶ τῆς «νοήσεως» εἶναι πράγματι «δόξαι» προσερχόμεναι ἀρχικῶς ἀπὸ τὴν ἐμπειρικὴν αἴσθησιν. Εἰς τὴν φάσιν τῆς «διανοίας» τὸ πνεῦμα δὲν δύναται νὰ προχωρήσῃ πέραν ἀπὸ τὰς ὑποθέσεις αὐτάς, ἀλλὰ χρησιμοποιεῖ ταύτας ὡς τμήματα γνώσεως, ἐφ' ὧν τοῦτο βασίζει ὅλας τοῦ τὰς ἐκδηλώσεις. Εἰς τὴν φάσιν τῆς «νοήσεως» τὸ πνεῦμα δύναται νὰ ὀδεύσῃ πέραν ἀπὸ αὐτάς τὰς ὑποθέσεις, διὰ νὰ συλλάβῃ τὰς μορφάς, αἵτινες ἐμπεριέχονται εἰς μίαν ἰδιαιτέραν διαλεκτικὴν ζήτησιν, Οὕτω ἡ μέθοδος τῆς «νοήσεως» ἥτις εἶναι ταυτόσημος μὲ τὴν «ἀνάβασιν» τῆς ὑποθετικῆς μεθόδου, ὡς περιγράφεται εἰς τὸν Φαίδωνα, εἶναι πράγματι προωρισμένη νὰ μετατρέπῃ τὰς «δόξας» ἢ τὰς ὑποθέσεις τῆς «διανοίας» εἰς γνῶσιν.

Ἡ «ἀνυπόθετος ἀρχή» δὲν εἶναι ἀναγκαίως τὸ ἀγαθὸν οὔτε μία πρότασις περὶ ἀγαθοῦ εἰς ὅλας τὰς διαλεκτικὰς ζητήσεις· εἶναι μᾶλλον μία πρότασις ὡς ἡ: «τί ἐκάνον» τοῦ Φαίδωνος — μία πρότασις ἥτις εἶναι ἐπαρκὴς ἐν τῇ ἐννοίᾳ ὅτι δὲν δύναται καθ' ἑαυτὴν νὰ ἐδραιωθῇ ὑπὸ μιᾶς ὑψηλοτέρας προτάσεως ἐντὸς τῶν ὁρίων τῆς ἰδιαιτέρας διαλεκτικῆς ζητήσεως. Ἐνῶ ἡ μελέτη τῶν διανοητικῶν μαθηματικῶν ἀρχίζει διὰ τοὺς φύλακας εἰς τὴν ἡλικίαν τῶν εἴκοσι, καὶ ἡ διαλεκτικὴ ἀρχίζει εἰς τὴν ἡλικίαν τῶν τριάκοντα, ἡ σπουδὴ τοῦ ἀγαθοῦ καθ' ἑαυτὴν ἀρχίζει εἰς τὴν ἡλικίαν τῶν πενήκοντα. Ἀποτελεῖ τὸ ἔσχατον (οὐχὶ τὸ πρῶτον) καὶ δυσκολώτατον πρᾶγμα, ὅπερ πρέπει νὰ γνωσθῇ εἰς τὸ βασιλείον τῆς γνώσεως — «ἐν τῷ γνωστῷ τελευταία ἡ τοῦ ἀγαθοῦ ἰδέα καὶ μόγις ὁρᾶσθαι» (517 b). Δηλαδή, ὁ Πλάτων εἰς τὴν πραγματικότητά δὲν λέγει εἰς τὴν Πολιτείαν ὅτι ὅλη ἡ γνῶσις συνάγεται τρόπον τινὰ ἀπὸ τὸ ἀγαθὸν ἢ εἶναι μία πρότασις περὶ ἀγαθοῦ.